

Matrices

PREFACE

We know a Matrix is a rectangular arrangement of numbers.

It is (i) rectangular and
(ii) arrangement

(i) Rectangular Means the number of Horizontal and Vertical line need not be the same.

Horizontal lines are called ROWS
and Vertical lines are called Columns

(ii) Arrangement Means any matrix has no specific value.

If a Matrix has m rows and n columns, it is a $m \times n$ Matrix. and denoted by $A = (a_{ij})_{m \times n}$

SUM

Sum and Product

We now define the Sum of two Matrices.

"If we want to add two Matrices The two must be of the same order

i.e. if A is of order $m \times n$ and B is also of order $m \times n$

then only can we add them

Thus if $A = (a_{ij})_{m \times n}$

$B = (b_{ij})_{m \times n}$

Then $A + B = (c_{ij})_{m \times n}$

s.t. $c_{ij} = a_{ij} + b_{ij}$

i.e. if $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$

and $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix}$

then $A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & a_{13} + b_{13} & a_{14} + b_{14} \\ a_{21} + b_{21} & a_{22} + b_{22} & a_{23} + b_{23} & a_{24} + b_{24} \\ a_{31} + b_{31} & a_{32} + b_{32} & a_{33} + b_{33} & a_{34} + b_{34} \\ a_{41} + b_{41} & a_{42} + b_{42} & a_{43} + b_{43} & a_{44} + b_{44} \end{bmatrix}$

If we multiply any matrix by any number, All the elements of the given matrix is Multiplied by that number.

i.e. if $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

Then $p.A = \begin{bmatrix} pa_{11} & pa_{12} & pa_{13} & pa_{14} \\ pa_{21} & pa_{22} & pa_{23} & pa_{24} \\ pa_{31} & pa_{32} & pa_{33} & pa_{34} \end{bmatrix}$

Thus $(-1)A = \begin{bmatrix} -a_{11} & -a_{12} & -a_{13} & -a_{14} \\ -a_{21} & -a_{22} & -a_{23} & -a_{24} \\ -a_{31} & -a_{32} & -a_{33} & -a_{34} \end{bmatrix}$

~~$A + (-A) = A - A = 0$~~

$= -A$ (also written as

We Can See $A + (-A) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$= 0$

So $A - A = 0$, the null matrix.

Associative Law of Addition

Let $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$ & $C = (c_{ij})_{m \times n}$

$$\begin{aligned}
 \text{Then } (A+B)+C &= \left\{ (a_{ij}) + (b_{ij}) \right\}_{m \times n} + (c_{ij})_{m \times n} \\
 &= \left\{ (a_{ij} + b_{ij}) \right\}_{m \times n} + (c_{ij})_{m \times n} \\
 &= \left\{ (a_{ij} + b_{ij}) + c_{ij} \right\}_{m \times n} \\
 &= \left\{ a_{ij} + (b_{ij} + c_{ij}) \right\}_{m \times n} \\
 &= (a_{ij})_{m \times n} + (b_{ij} + c_{ij})_{m \times n} \\
 &= A + (B + C)
 \end{aligned}$$

So Matrix Addition is Associative

Commutative Law

Let $A = (a_{ij})_{m \times n}$ & $B = (b_{ij})_{m \times n}$

Then $A+B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n}$

$$= (a_{ij} + b_{ij})_{m \times n}$$

$$= (b_{ij} + a_{ij})_{m \times n}$$

$$= (b_{ij})_{m \times n} + (a_{ij})_{m \times n}$$

$$= B + A. \text{ So Addition is commutative.}$$

We have seen that

if A is a $m \times n$ matrix
and we have a null matrix of
order $m \times n$ then

$A + [0] = A$ so we ~~are~~
Identity element w.r. to addition
exists

Further ~~of Identity matrix~~
 $m \times n$

for any given matrix $A = (a_{ij})_{m \times n}$

we have $-A = (-1)A$ is a matrix

$$\text{s.t. } A + (-A) = [0]_{m \times n}$$

So the Additive Inverse of any
matrix exists.