Matrix Product

If $A=(a_{ij})$ is a $m\times n$ matrix and $B=(b_{ij})$ is a $p\times q$ matrix and we want to multiply A and B i.e. find $A\times B$ then we must have $n=p$. i.e. ~~we must~~ To find $A\times B$ we must have that number of columns in A is the same as the number of rows of B and in such a case the order of the product i.e. the order of $A\times B$ will be $m\times q$.

Further, if $A\times B = C=(C_{rt})$ then
$$C_{rt} = \sum_{k=1}^{n} a_{rk} b_{kt}$$
$$= a_{r1}b_{1t} + a_{r2}b_{2t} + a_{r3}b_{3t} + \cdots + a_{rn}b_{nt}$$

Example

Let $A=\begin{bmatrix} a & b & c \\ p & q & r \end{bmatrix}$ is a 2×3 matrix

If we want to multiply it with another matrix B, then B must have 3 rows. The number of columns in B does not matter.

Let $B=\begin{bmatrix} 1 & 4 & 3 \\ 2 & 6 & 4 \\ 3 & 9 & 2 \end{bmatrix}$ a 3×3 Matrix

Then $A\times B = \begin{bmatrix} a.1+b.2+c.3 & a.4+b.6+c.9 & a.3+b.4+c.2 \\ p.1+q.2+r.3 & p.4+q.6+r.4 & p.3+q.4+r.2 \end{bmatrix}$

Scanned with CamScanner

i.e. $A \times B = \begin{bmatrix} a+2b+3c & 4a+6b+9c & 3a+4b+2c \\ p+2q+3r & 4p+6q+9r & 3p+4q+2r \end{bmatrix}$

which is a (2×3) matrix.

We now try to find $B \times A$

In this case a B is 3×3 matrix while A is 2×3 matrix and So we can not find $B \times A$

If we take $A = \begin{bmatrix} a & b \\ x & y \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

then $A \times B = \begin{bmatrix} a+3b & 2a+4b \\ x+3y & 2x+4y \end{bmatrix}$

while $B \times A = \begin{bmatrix} a+2b & b+2y \\ 3a+4x & 3b+4y \end{bmatrix}$

obviously $A \times B \neq B \times A$ here

Thus we say that even if both $A \times B$ and $B \times A$ be calculated then in general $A \times B \neq B \times A$

i.e. Matrix Multiplication is not Commutative.

Associative

Let $A = (a_{ij})$ is a $m \times n$ Matrix
~~&~~ $B = (b_{jk})$ is a $n \times p$ Matrix
and $C = (c_{kl})$ is a $p \times q$ Matrix

We want to check if $A \times (B \times C) = (A \times B) \times C$
For this we consider the element ~~pr^{th} or~~ in the p^{th} row and r^{th} column on ~~both~~ sides.

For the left side,

$r\alpha^{th}$ term of $B \times C$

$$= \sum_{k} b_{rk} c_{k\alpha}$$

So, pr^{th} term of $A \times (B \times C)$ is

$$= \sum_{t} a_{pt} \left(\sum_{\alpha} (b_{t\alpha} c_{\alpha t}) \right)$$

Similarly for Right Side, the pq^{th} term of $A \times B$ is $\sum_{\beta} a_{p\beta} b_{\beta q}$ and thus

the r^{th} term of $(A \times B) \times C = \sum_{r} \left(\sum_{\beta} a_{p\beta} b_{\beta r} \right) c_{rq}$

Thus both terms are the same and

So we get $A \times (B \times C) = (A \times B) \times C$
Thus Matrix Multiplication is Associative.

~~and So~~ if $A \times (B \times C)$ and ~~$A \times B \times$~~ $(A \times B) \times$
~~and~~ so $A \times (B \times C) = A$

Extending this Associative Law we see $A\times(A\times A) = (A\times A)\times A$

$= A^3$

Similarly $(A\times A)\times(A\times A) = A^4$

and so on.

~~For~~

Distributive Law.

~~$A\times(B+C) = A\times[$~~

Here, ~~take~~ $A=(a_{ij})$, a $m\times n$ matrix.

$B=(b_{pq})$ a $n\times q$ matrix

~~and~~ $C=(C_{pq})$ a $n\times q$ matrix

So $B+C = (b_{pq}+c_{pq})$ a $n\times q$ Matrix

$\therefore A\times(B+C) = \sum_{p=1}^{n} a_{ip}(b_{pq}+c_{pq})$

$= \sum_{p} (a_{ip}b_{pq} + a_{ip}c_{pq})$

$= \sum_{p} a_{ip}b_{pq} + \sum_{p} a_{ip}c_{pq}$

$= A\times B + A\times C$

~~i.e. $A\times(B+C) = A\times B + A\times C$~~

Thus Distributive Law Holds.